DIFFERENTIAL CALCULUS

1. Let $f(x) = \{b^2 + (a-1)b - 1/4\}x + \int_0^x (\sin^4 x + \cos^4 x)dx$. If $f(x)$ be a non-decreasing function of x, $\forall x \in R$ and $\forall b \in R$. Then the number of positive integer values of parameter a is ____

Sol:

1. (2) $f(x) = \{b^2 + (a-1)b - 1/4\} + (\sin^4 x + \cos^4 x)$

Since function is non-decreasing

$f(x) \geq 0 \forall x \in R$ and $\forall b \in R$

$\Rightarrow \sin^4 x + \cos^4 x \geq -b^2 - (a-1)b + 1/4$

$\Rightarrow -b^2 - (a-1)b + 1/4 \leq (\sin^4 x + \cos^4 x)$

But minimum value of $\sin^4 x + \cos^4 x$ is $\frac{1}{2}$

Thus from equation (i)

$-b^2 - (a-1)b + \frac{1}{4} \leq \frac{1}{2} \forall b \in R$

$b^2 + (a-1)b + \frac{1}{4} \geq 0 \forall b \in R$

Now, $D \leq 0$

$\Rightarrow (a-1)^2 - 4(1)(1/4) \leq 0$

$\Rightarrow (a-1)^2 - 1 \leq 0 \Rightarrow a \in [0, 2]$

2. $f(x)$ and $g(x)$ are polynomials of degree 3. $g(\alpha) = g'(\alpha) = 0$. If $\lim_{x \to \alpha} \frac{f(x)}{g(x)} = 0$, then find the number of different solutions of equation $f(x)g'(x) + f'(x)g(x) = 0$.

2. (2) For existence of the given limit $f(\alpha) = f'(\alpha) = f''(\alpha) = 0$

$\because f(x)$ is polynomial of degree $3 \Rightarrow f(x) = \lambda(x-\alpha)^3$

Similarly $g(x) = \mu(x-\alpha)^2(x-\beta)$ $(\beta \neq \alpha)$

$\Rightarrow f(x)g(x) = \lambda\mu(x-\alpha)^5(x-\beta)$

$\therefore$ Number of different solution of $\frac{d}{dx}(f(x) \cdot g(x)) = 0$ are 2 i.e. one solution α and one between α and β.

Integral cal